Cross Classification

Factor B	Factor A		
	1	2	3
1	a_1b_1	a_2b_1	a_3b_1
2	a_1b_2	a_2b_2	a_3b_2
3	a_1b_3	a_2b_3	a_3b_3

All levels of Factor A occur in combination with all levels of Factor B.

Nested Factors

B Nested Within A

Factor B	Factor A		
	1	2	3
1	a_1b_1	a_2b_4	a_3b_7
2	a_1b_2	a_2b_5	a_3b_8
3	a_1b_3	a_2b_6	a_3b_9

Different levels of Factor B occur within each level of Factor A.

Nested Factors

A Familiar Example – One-Way ANOVA

$$Y_{ij} = \mu + T_i + \varepsilon_{(i)j}$$

Treatments	**Plots**			
1	1	2	3	4
2	5	6	7	8
3	9	10	11	12
4	13	14	15	16

- No two plots are exactly the same
- ∴ plots are nested in treatments
- The parentheses around i subscript in the error term indicate that j (plots) is nested within i (treatments)

Nested Factors

B Nested Within A

Linear Additive Model:

$$Y_{ijk} = \mu + A_i + B_{(i)j} + \varepsilon_{(ij)k}$$

Where:

- Y_{ijk} = varaible to be analyzed from the k^{th} experimental unit
- μ = overall mean
- A_i = effect of the i^{th} level of A
- $B_{(i)j}$ = effect of the j^{th} level of B within the i^{th} level of A
- $\varepsilon_{(ij)k}$ = experimental error associated with k^{th} experimental unit, $NID(0,s^2)$

Nested Factors
ANOVA

Degrees of Freedom:

Factor A	(I - 1)
Factor B within A	I(J - 1)
Error	IJ(K - 1)

Nested Factors
ANOVA

Sums of Squares:

Factor A

(I - 1)

$(\bar{y}_{i..} - \bar{y}_{...})^2$

$$SS(A) = JK\sum_{i=1}^{I}(\bar{y}_{i..} - \bar{y}_{...})^2$$

Nested Factors

ANOVA

Sums of Squares:

Factor B within A

I(J - 1) = IJ - I

$(\bar{y}_{ij.} - \bar{y}_{i..})^2$

$$SS(B) = K\sum_{i=1}^{I}\sum_{j=1}^{J}(\bar{y}_{ij.} - \bar{y}_{i..})^2$$

Nested Factors

ANOVA

Sums of Squares:

Error

IJ(K - 1) = IJK - IJ

$(y_{ijk} - \bar{y}_{ij.})^2$

$$SS(error) = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(y_{ijk} - \bar{y}_{ij.})^2$$

Nested Factors

Relationship Between Nested and Factorial SS

```
MODEL Y = plant leaf(plant);
Source          DF        Anova SS
plant            3      7.56034583
leaf(plant)      8      2.63020000

MODEL Y = plant | leaf;
Source          DF        Anova SS
plant            3      7.56034583
leaf             2      1.21105833
plant*leaf       6      1.41914167
```

SS leaf(plant) = SS leaf + SS plant*leaf = 1.211 + 1.419 = 2.630

df leaf(plant) = df leaf + df plant*leaf = 2 + 6 = 8

Nested Factors

ANOVA

<u>Mean Squares:</u>

MS(A) = SS(A) / (I - 1)

MS(B/A) = SS(B/A) / I(J -1)

MS(error) = SS(error) / [IJ(K - 1)]

Nested Factors
Expected Mean Squares

Model: $Y_{ijk} = \mu + A_i + B_{(i)j} + \varepsilon_{(ij)k}$

Source	a F i	b R j	r R k	EMS
A_i	0	b	r	
$B_{(i)j}$	1	1	r	
$\varepsilon_{(ij)k}$	1	1	1	

Nested Factors
Expected Mean Squares

Model: $Y_{ijk} = \mu + A_i + B_{(i)j} + \varepsilon_{(ij)k}$

Source	a F i	b R j	r R k	EMS
A_i	0	b	r	$\sigma^2 + r\sigma^2_B + br\Phi(A)$
$B_{(i)j}$	1	1	r	$\sigma^2 + r\sigma^2_B$
$\varepsilon_{(ij)k}$	1	1	1	σ^2

Lorenzen and Anderson, Problems 4.5 & 4.11

$Y_{ijklm} = \mu + A_i + B_{(i)j} + C_{(ij)k} + D_l + AD_{il} + BD_{(i)jl} + CD_{(ij)kl} + E_{(l)m} + AE_{i(l)m} + BE_{(il)jm} + CE_{(ijl)km}$

Source	df	2 F i	2 R j	2 R k	2 R l	2 R m	EMS
A_i	1	0	2	2	2	2	
$B_{(i)j}$	2	1	1	2	2	2	
$C_{(ij)k}$	4	1	1	1	2	2	
D_l	1	2	2	2	1	2	
AD_{il}	1	0	2	2	1	2	
$BD_{(i)jl}$	2	1	1	2	1	2	
$CD_{(ij)kl}$	4	1	1	1	1	2	
$E_{(l)m}$	2	2	2	2	1	1	
$AE_{i(l)m}$	2	0	2	2	1	1	
$BE_{(il)jm}$	4	1	1	2	1	1	
$CE_{(ijl)km}$	8	1	1	1	1	1	

Lorenzen and Anderson, Problems 4.5 & 4.11

$Y_{ijklm} = \mu + A_i + B_{(i)j} + C_{(ij)k} + D_l + AD_{il} + BD_{(i)jl} + CD_{(ij)kl} + E_{(l)m} + AE_{i(l)m} + BE_{(il)jm} + CE_{(ijl)km}$

Source	df	2 F i	2 R j	2 R k	2 R l	2 R m	EMS
A_i	1	0	2	2	2	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 4\sigma^2_{AE} + 2\sigma^2_{CD} + 4\sigma^2_{BD} + 8\sigma^2_{AD} + 4\sigma^2_C + 8\sigma^2_B + 16\Phi[A]$
$B_{(i)j}$	2	1	1	2	2	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 2\sigma^2_{CD} + 4\sigma^2_{BD} + 4\sigma^2_C + 8\sigma^2_B$
$C_{(ij)k}$	4	1	1	1	2	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{CD} + 4\sigma^2_C$
D_l	1	2	2	2	1	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 8\sigma^2_E + 2\sigma^2_{CD} + 4\sigma^2_{BD} + 16\sigma^2_D$
AD_{il}	1	0	2	2	1	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 4\sigma^2_{AE} + 2\sigma^2_{CD} + 4\sigma^2_{BD} + 8\sigma^2_{AD}$
$BD_{(i)jl}$	2	1	1	2	1	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 2\sigma^2_{CD} + 4\sigma^2_{BD}$
$CD_{(ij)kl}$	4	1	1	1	1	2	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{CD}$
$E_{(l)m}$	2	2	2	2	1	1	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 8\sigma^2_E$
$AE_{i(l)m}$	2	0	2	2	1	1	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE} + 4\sigma^2_{AE}$
$BE_{(il)jm}$	4	1	1	2	1	1	$\sigma^2 + \sigma^2_{CE} + 2\sigma^2_{BE}$
$CE_{(ijl)km}$	8	1	1	1	1	1	$\sigma^2 + \sigma^2_{CE}$

Nested Factors

Expected Mean Square Rules

The EMS for each model term consists of:

- σ^2
- a variance component associated with the term
- a variance component associated with each interaction with the term where 1) all other factors are random (sum-to-zero model) **or** 2) any mixed interaction that contains the term (independence model)
- a variance component for factors nested within the term

Coefficients:

- for σ^2 is 1
- for all other components is equal to the product of all treatment levels not included in the main effect or interaction
- for nested components is equal to the product of all treatment levels not included in the component

Nested Factors

EMS Rules Example

Model: Y = A B(A) C AC B(A)C

A and C fixed, B random

A	
B(A)	
C	
AC	
B(A)C	

Nested Factors

EMS Rules Example

Model: Y = A B(A) C AC B(A)C

A and C fixed, B random

A	$\sigma^2 + c\sigma^2_{B(A)} + bc\Phi(A)$
B(A)	$\sigma^2 + c\sigma^2_{B(A)}$
C	$\sigma^2 + \sigma^2_{B(A)C} + ab\Phi(C)$
AC	$\sigma^2 + \sigma^2_{B(A)C} + b\Phi(AC)$
B(A)C	$\sigma^2 + \sigma^2_{B(A)C}$

Nested Factors

Agronomic Example

<u>Factors</u>:

Region (R)

State (S)

Variety (V)

Nested Factors
Agronomic Example

Layout:

	Region					
	Midwest			Great Plains		
State	IA	IL	IN	NE	KS	SD
Variety	1	11	21	31	41	51
	2	.	.	.	.	.
	3	.	.	.	.	.
	4	.	.	.	.	.
	5	.	.	.	.	.
	6	.	.	.	.	.
	7	.	.	.	.	.
	8	.	.	.	.	.
	9	.	.	.	.	.
	10	.	.	.	.	60

Nested Factors
Agronomic Example

Model:

$$Y_{ijk} = \mu + R_i + S_{(i)j} + V_{(ij)k} + \varepsilon_{(ijk)}$$

Where:

R_i = effect of i^{th} region

$S_{(i)j}$ = effect of j^{th} state within i^{th} region

$V_{(ij)k}$ = effect of the k^{th} variety within the j^{th} state within i^{th} region

Nested Factors

Agronomic Example

Expected Mean Squares:

Source	r F i	s R j	v R k	EMS
R_i	0	s	v	$\sigma^2 + \sigma^2_V + v\sigma^2_S + sv\Phi(R)$
$S_{(i)j}$	1	1	v	$\sigma^2 + \sigma^2_V + v\sigma^2_S$
$V_{(ij)k}$	1	1	1	$\sigma^2 + \sigma^2_V$
$\sigma_{(ijk)}$	1	1	1	σ^2

Nested Factors

Seed Experiment Example

Factors:

State (S)	4	F
Variety (V)	2	F
Lot (L/V)	3	R
Treatment	3	F

Nested Factors

Seed Experiment Example

Layout:

State	1						2						3						4					
Variety	1			2			1			2			1			2			1			2		
Lot(V)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Treatment	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Linear Additive Model:

$$Y_{ijkl} = \mu + S_i + V_j + SV_{ij} + L_{(ij)k} + T_l + ST_{il} + VT_{jl} + SVT_{ijl} + LT_{(ij)kl}$$

Nested Factors

Seed Experiment Example

Expected Mean Squares

Source	df	4 F i	2 F j	3 R k	3 F l	EMS
S_i	3	0	2	3	3	$3\sigma^2_L + 18\Phi(S)$
V_j	1	4	0	3	3	$3\sigma^2_L + 36\Phi(V)$
SV_{ij}	3	0	0	3	3	$3\sigma^2_L + 9\Phi(SV)$
$L_{(ij)k}$	16	1	1	1	3	$3\sigma^2_L$
T_l	2	4	2	3	0	$\sigma^2_{LT} + 24\Phi(T)$
ST_{il}	6	0	2	3	0	$\sigma^2_{LT} + 6\Phi(ST)$
VT_{jl}	2	4	0	3	0	$\sigma^2_{LT} + 12\Phi(VT)$
SVT_{ijl}	6	0	0	3	0	$\sigma^2_{LT} + 3\Phi(SVT)$
$LT_{(ij)kl}$	32	1	1	1	0	σ^2_{LT}

Nested Factors

Seed Experiment Example

SAS Code

```
proc glm;
  class state var lot trt;
  model germ = state var state*var lot(state*var)
     trt state*trt var*trt state*var*trt);
  test h=state var state*var e=lot(state*var);
run;
```

Specifying Nested Effects:

- Use parentheses to indicate nesting of treatment factors
- Place factor in which nesting occurs within parentheses immediately after nested factor with no space between
- Corresponds to subscripts in linear model: $L_{(ij)k}$

Fixed Factors

That Appear Nested

Chemical	Rate	g/ha
Kill Dead	.5x	200
	1x	400
	1.5x	600
Roadkill	.5x	2k
	1x	4k
	1.5x	6k

Fixed Factors
That Appear Nested

```
proc glm;
  class chem rate;
  model weed = chem | rate;
  lsmeans chem*rate / slice=chem;
run;
```

Sliding Factors
Example

Row Spacing	Population	plants / acre
30 in.	low	20k
	med.	25k
	high	30k
45-in.	low	15k
	med.	20k
	high	25k

Sliding Factors
Example

Treatment	Row Spacing	Population
1	30 in.	20k
2	30 in.	25k
3	30 in.	30k
4	45-in.	15k
5	45-in.	20k
6	45-in.	25k

Sliding Factors
Example

```
proc glm;
  class trt;
  model yld = trt;
  contrast 'row,overall'trt 1 1 1 -1 -1 -1;
  contrast '15v30k'trt 0 0 1 -1 0 0;
  contrast 'row,20-25k'trt 1 1 0 0 -1 -1;
  contrast 'pop,20v25k'trt 1 -1 0 0 1 -1;
  contrast 'row*pop,20-25k'trt 1 -1 0 0 -1 1;
run;
```